NEUTRINO MASS AND GRAND UNIFICATION OF FLAVOR

R. N. MOHAPATRA*

Maryland Center for Fundamental Physics,

University of Maryland,

Maryland, MD-20742, USA

*E-mail: rmohapat@umd.edu

Abstract

The problem of understanding quark mass and mixing hierarchies has been an outstanding problem of particle physics for a long time. The discovery of neutrino masses in the past decade, exhibiting mixing and mass patterns so very different from the quark sector has added an extra dimension to this puzzle. This is specially difficult to understand within the framework of conventional grand unified theories which are supposed to unify the quarks and leptons at short distance scales. In the paper, I discuss a recent proposal by Dutta, Mimura and this author that appears to provide a promising way to resolve this puzzle. After stating the ansatz, we show how it can be realized within a SO(10) grand unification framework. Just as Gell-Mann's suggestion of SU(3) symmetry as a way to understand the hadronic flavor puzzle of the sixties led to the foundation of modern particle physics, one could hope that a satisfactory resolution of the current quark-lepton flavor problem would provide fundamental insight into the nature of physics beyond the standard model.

I. INTRODUCTION

The quark masses as well as their mixings exhibit a hierarchical pattern i.e. for masses $m_{u,d} \ll m_{c,s} \ll m_{t,b}$; for mixing angles $V_{ub} \ll V_{cb} \ll V_{cd} \ll V_{ud,cs,tb}$. This is known as the flavor puzzle for quarks. Unravelling this puzzle has long been recognized as a challenge for physics beyond the standard model [1].In the 1960's, a puzzle of similar nature was the focus of attention of many when particle physicists tried to understand why there were different baryons and mesons with masses close to each other. This puzzle, the flavor puzzle of the sixties was solved in a seminal paper by Prof. Gell-Mann- the famous "Eight-fold Way" paper, that practically the first read for every graduate student aspiring to be a particle theorist in the sixties. In this paper, he proposed that there is an underlying symmetry for hadrons interactions, the SU(3) symmetry which is responsible for the closeness of observed mass and quantum number patterns. This led to the so called Gell-Mann-Okubo mass formula, which was extremely successful in understanding the baryon and meson spectra and it culminated with the discovery of the Ω^- meson by N. Samios et al. This proposal of Gell-Mann unleased an idea that had a profound impact on particle physics: it led to the concepts of quarks as the constituents of hadrons which forms the foundation of modern particle physics. It subsequently led to the birth of Quantum Chromodynamics as the theory of strong forces etc.

In the modern day particle physics of quarks and leptons, the hope has always been that solving the flavor puzzle may have similar ground breaking implication for physics beyond the standard model.

The puzzle of quark flavors was already mysterious but it got more so after the discovery of neutrino masses and mixings in 1998 and subsequent years. Unlike the quark mixings, lepton mixings do not exhibit a hierarchical pattern (i.e. neutrino mixings between generations denoted by θ_{ij} are given by $\theta_{23}^l \sim 45°$ and $\theta_{12}^l \simeq 35°$ as against $\theta_{23}^q \sim 2.5°$ and $\theta_{12}^q \sim 13°$) and the neutrino masses also do not exhibit as strong a hierarchy as quarks or charged leptons i.e. for a normal hierarchy for neutrinos, $m_2/m_3 \simeq V_{us} \ll m_\mu/m_\tau$. In fact the lepton mixing matrix (the PMNS matrix) appears to very closely resemble the following pattern: called the tri-bi-maximal mixing matrix[2]:

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{1}$$

On top of this, neutrinos being electrically neutral particles could be their own anti-particles, the so-called Majorana fermions. In fact, the most popular way to understand the small neutrino masses seems to be the seesaw mechanism[3], which predicts that the neutrinos are

their own anti-particles. It could be that the different mixing pattern for leptons is related to this feature. Our proposal below has this as one of its ingredients.

The problem of quark-lepton masses and mixings becomes specially puzzling in grand unified theories where the quarks and leptons unify at a very high scale. One would naively expect that when quarks and leptons unify, their masses and mixings would exhibit a similar pattern. In fact the seesaw mechanism also suggests that the scale of neutrino mass is about 10^{14} GeV which is very close to the conventional grand unification scale $\sim 10^{16}$ GeV or so. This then raises the fundamental question of how we understand the diverse pattern of quarks and leptons in a seesaw motivated grand unified theory. Cracking this code may provide a hint of some really new exciting underlying physics.

In this note, I describe a recent ansatz proposed by Dutta and Mimura and this author which promises a new way to have a unified understanding of quark lepton flavor[4] and its realization in SO(10) grand unified theories .

As a prelude to this discussion, let us realize that one way to understand the quark mass hierarchy is to start with a rank one mass matrix for up, down quarks and charged leptons in the leading order i.e.

$$M_{u,d,l} \;=\; m_{t,b,\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

and consider corrections coming from non-leading operators. Second clue for our ansatz is the observation that small quark mixings are an indication that one could write the up and down quark mass matrices as a sum a "big" matrix plus a small matrix with the "big" matrix part for both sectors being proportional to each other so that in the leading order the CKM angles vanish e.g.

$$M_{u,d} \;=\; M^0_{u,d} + \delta_{u,d} \tag{3}$$

with $M^0_d = r M^0_u$ being the "big" matrix and $\delta_{u,d}$ being the smaller part. As just mentioned, the proportionality of the large parts of the mass matrices guarantees that the mixing angles will necessarily be small since the diagonalizing matrix for the large parts are "parallel" or "aligned" and the nontrivial CKM matrix represents the "small" misalignment between the two matrices determined by the "smaller" parts of the mass matrix.

We can therefore now state our ansatz[4] which consists of two parts:

1. The quark and lepton mass matrices have the following general feature:

$$M_u \;=\; M_0 \;+\; \delta_u; \tag{4}$$

3

$$M_d = rM_0 + \delta_d;$$
$$M_l = rM_0 + \delta_l;$$
$$M_\nu = fv_L$$

2. M_0 has rank one.

Note that by a choice of the lepton basis, we can make f diagonal without loss of generality. It is then clear from the first part of the ansatz (item one) that for an "anarchic" form of the matrices M_0 and δ as long as $\delta_{ij} \ll M_0$, the lepton mixing angles are large whereas the quark mixing angles are small. The second rank one property than guarantees that quark and charged lepton masses are hierarchical whereas since f matrix is arbitrary, any hierarchy in the neutrino sector is likely to be milder. Incidentally, rank one property to understand mass hierarchy has been used in the past; see for instance[5].

II. GAUGE GROUP REQUIRED TO IMPLEMENT THE ANSATZ

The question that arises next is how to implement our ansatz with a gauge model framework. To implement the first part, it is important to notice that the up and down quark mass matrices must be proportional to each other. Such relations do not emerge from the standard model since the u_R and d_R fields are separate fields and their Yukawa couplings responsible for the up and down quark mass matrices are therefore independent of each other. The situation however changes once we expand the gauge group to the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group since the (u_R, d_R) form a doublet of the $SU(2)_R$ group and thus relate the up and down Yukawa matrices[6]. Since quark-lepton unifications arises naturally within grand unification theories, the obvious group to consider is the SO(10) group as we do in the next section, although the basic conditions of the ansatz could also be realized with less predictive power within the $SU(2)_L \times SU(2)_R \times SU(4)_c$ partial unification groups.

III. SO(10) REALIZATION OF FLAVOR UNIFICATION ANSATZ

As is well known, in SO(10) models, the matter fermions belong to **16**-dim. spinor representations. To get fermion masses, we will consider SO(10) models with **10**, **126** plus possibly another **10** or **120** Higgs fields where fermion masses are generated by renormalizable Yukawa couplings [7] only and where type II seesaw[8] is responsible for neutrino masses [9]. To implement our idea, we require that one of the **10** Yukawa couplings is the dominant one contributing to up, down and charged lepton masses and has rank one with

other smaller couplings providing neutrino masses as well as most of the quark lepton flavor hierarchy. We postpone the discussion of how to get rank one till later. Let us see if this model does indeed give us our ansatz. It is well known that in these models[7], we have the following form for all fermion masses:

$$
\begin{aligned}
Y_u &= h + r_2 f + r_3 h', \\
Y_d &= r_1 (h + f + h'), \\
Y_e &= r_1 (h - 3f + c_e h'), \\
Y_{\nu D} &= h - 3r_2 f + c_\nu h',
\end{aligned}
\tag{5}
$$

where Y_a are mass matrices divided by the electro-weak vev v_{wk} and r_i and $c_{e,\nu}$ are the mixing parameters which relate the $H_{u,d}$ to the doublets in the various GUT multiplets. More precisely, the matrices h, f and h' in Y_a are multiplied by the Higgs mixing parameters when they appear in the fermion mass matrices.

Furthermore, we use the type II seesaw formula[8] for getting neutrino masses gives [9].

$$
\mathcal{M}_\nu = f v_L.
\tag{6}
$$

Note that f is the same coupling matrix that appears in the charged fermion masses in Eq. (5), up to factors from the Higgs mixings and the Clebsch-Gordan coefficients. This helps us to connect the neutrino parameters to the quark-sector parameters. The equations (5) and (6) are the key equations in our unified approach to addressing the flavor problem and obviously satisfy our flavor unification ansatz.

IV.   IMPLEMENTING RANK ONE STRATEGY

The rank one Yukawa coupling with **10** Higgs field generates the features of flavor hierarchy, and rank 1 matrices can often appear in various ways (flavor symmetry, discrete symmetry, and string models). In this section, we give an SO(10) model, where the rank one ansatz used in our discussion of flavor emerges from extra vector like spinors above the GUT scale as well as a discrete symmetry.

When the direct couplings of chiral fermions with a Higgs field are forbidden by the chosen discrete symmetry, and the effective Yukawa couplings are generated by propagating vector-like matter fields, the rank of the effective Yukawa matrix depends on the number of the vector-like fields. Actually, when there are only one pair of vector-like matter fields as a flavor singlet, the effective Yukawa matrix is rank 1.

To illustrate this in a warm-up example, we consider a model which has one extra vector-like pair of matter fields to start with with mass slightly above the GUT scale contributing

to the **10** coupling (denoted by $\psi_V \equiv \mathbf{16}_V \oplus \bar{\psi}_V \equiv \overline{\mathbf{16}}_V$) and three gauge singlet fields Y_a. We add a Z_4 discrete symmetry to the model under which the fields $\psi_a \to i\psi_a$, and $Y_a \to -iY_a$. The **10**-Higgs field H is invariant under this symmetry. The gauge invariant Yukawa superpotential under this assumption is given by

$$W = \psi_V H \lambda \psi_V + M_V \psi_V \bar{\psi}_V + \bar{\psi}_V \sum_a Y_a \psi_a. \tag{7}$$

When we give vevs $\langle Y_a \rangle \neq 0$, ψ_V and ψ_a are mixed. The heavy vector-like fields, $\bar{\psi}_V$ and a linear combination of ψ_V and ψ_a (i.e. $M_V \psi_V + \sum_a Y_a \psi_a$), and the effective operator below its scale and at the GUT scale is given by:

$$\mathcal{L}_{eff} = \frac{\lambda}{M_V^2 + \sum_a Y_a^2} \left[\sum_a Y_a \psi_a \right] H \left[\sum_b Y_b \psi_b \right]. \tag{8}$$

This gives rise to a rank one h coupling. We note that it does not contradict the $O(1)$ top Yukawa coupling, when $M_V^2 \sim \sum_a Y_a^2$ (or $M_V^2 < \sum_a Y_a^2$).

If we let the $\overline{\mathbf{126}}$ Higgs field transform like -1 under Z_4, it can induce the f coupling with rank three. Our final model given below builds on this but differs in details e.g. it has got two vectorlike spinor multiplets instead of one etc.

V. MAKING THE MODEL PREDICTIVE

Simply using the above ansatz in the context of an SO(10) model with mass relations in Eq. (5), turns out to reproduce the qualitative features of the quark and lepton spectra quite well. For example in the context of a two generation model (involving the second and third generation), this simple ansatz predicts $V_{cb} \simeq (m_s/m_b + e^{i\sigma} m_c/m_t) \cot\theta$, where θ is the atmospheric mixing angle. This relation is in rough agreement with observations to leading order. In addition, we have at GUT scale $m_b \sim m_\tau$ as well as $m - \mu \sim -3m_s$ also in rough agreement with observations.

Encouraged by these results, we can be more ambitious and start using this ansatz in combination of other ideas to make as many predictions as possible. To this end, we note that in the limit vanishing Yukawa couplings, the standard model has $[U(3)]^5$ global symmetry. It is therefore quite possible that in the final understanding of flavor, a subgroup of this large symmetry does play an important role[10], specially subgroups which have three dimensional representation to fit three generations. In order to exploit this observation, one may replace all Yukawa couplings by flavon fields which transform as three dimensional representations of a subgroup of $[SU(3)]^5$ and consider the minima of the flavon theory in flavon space as determining the values of the Yukawa couplings. It turns out that there are nontrivial

	ψ	H	$\bar{\Delta}$	H'	ϕ_1	$\bar{\phi}_1$	ϕ_2	$\bar{\phi}_2$	ϕ_3	$\bar{\phi}_3$	ψ_{V1}	$\bar{\psi}_{V1}$	ψ_{V2}	$\bar{\psi}_{V2}$
SO(10)	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{126}}$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{16}$	$\overline{\mathbf{16}}$	$\mathbf{16}$	$\overline{\mathbf{16}}$
S_4	$\mathbf{3_2}$	$\mathbf{1_1}$	$\mathbf{1_2}$	$\mathbf{1_1}$	$\mathbf{3_2}$	$\mathbf{3_2}$	$\mathbf{3_1}$	$\mathbf{3_1}$	$\mathbf{3_2}$	$\mathbf{3_2}$	$\mathbf{1_1}$	$\mathbf{1_1}$	$\mathbf{1_2}$	$\mathbf{1_2}$
Z_n	1	ω^{-4}	ω^{-2-a}	ω^{-1}	ω^2	ω^{-2}	ω	ω^{-1}	ω^{α}	$\omega^{-\alpha}$	ω^2	ω^{-2}	ω	ω^{-1}

examples where this program is realized. In the second paper of [4], we presented an S_4 subgroup example. Below I briefly recapitulate this example.

VI.   THE $SO(10) \times S_4 \times Z_n$ MODEL OF FLAVOR

Recall that the S_4 group is a 24 element group describing permutations of four distinct objects and has five irreducible representations with dimensions $\mathbf{3_1} \oplus \mathbf{3_2} \oplus \mathbf{2} \oplus \mathbf{1_2} \oplus \mathbf{1_1}$. The distinction between the representations with subscripts 1 and 2 is that the later change sign under the transformation of group elements involving the odd number of permutations of S_4.

We assign the three families of $\mathbf{16}$-dim. matter fermions ψ to $\mathbf{3_2}$-dim. representation of S_4 and the Higgs field H, $\bar{\Delta}$ and H' to $\mathbf{1_1}, \mathbf{1_2}$, and $\mathbf{1_1}$ reps, respectively. We then choose three SO(10) singlet flavons ϕ_i transforming as $\mathbf{3_2}, \mathbf{3_1}, \mathbf{3_2}$ reps of S_4 and one gauge and S_4 singlet fields s_1, s_2 transforming as $\mathbf{1_2}$ and $\mathbf{1_1}$ respectively. We further assume that at a scale slightly above the GUT scale, there are two S_4 singlet vectorlike pairs of $\mathbf{16} \oplus \overline{\mathbf{16}}$ fields denoted by ψ_V and $\bar{\psi}_V$. In order to get the desired Yukawa couplings naturally from this high scale theory, we supplement the S_4 group by an Z_n group with all the above fields belonging to representations given in the Table I. The fields and representations to generate the desired Yukawa couplings. $\omega = e^{i\frac{2\pi}{n}}$. $\alpha = 2 + a - b$. In addition to the fields below, there are two gauge and S_4 singlet $1_2, 1_1$ fields with Z_n quantum numbers ω^a and ω^b respectively.

The most general high scale Yukawa superpotential involving matter fields invariant under this symmetry is given by:

$$W = (\phi_1 \psi)\bar{\psi}_{V1} \; + \; \psi_{V1}\psi_{V1}H \; + \; M_1 \bar{\psi}_{V1}\psi_{V1} \tag{9}$$
$$+ (\phi_2 \psi)\bar{\psi}_{V2} \; + \; \frac{1}{M_P}s_1 \psi_{V2}\psi_{V2}\bar{\Delta} + \; M_2 \bar{\psi}_{V2}\psi_{V2}$$
$$+ \frac{1}{M_P^2}s_2(\phi_3 \psi\psi)\bar{\Delta} + \frac{1}{M_P}(\phi_2 \psi\psi)H',$$

where the brackets stand for the S_4 singlet contraction of flavor index. The singlet field s_i can have large vev as follows: consider its Z_n charge to be such that the only polynomial term involving the s_i in the superpotential has the form $s_i^{k_i}/M_P^{k_i-3}$ (in order to describe the

essential potential, we ignore a possible $s_1^{\ell_1} s_2^{\ell_2}$ term). The dominant part of the potential in the presence of SUSY breaking has the form:

$$V(s_i) \;=\; -m_{s_i}^2 |s_i|^2 + k \frac{s_i^{2k_i-2}}{M_P^{2k_i-6}} + \cdots . \tag{10}$$

Minimizing this leads to $\langle s_i \rangle \sim [m_{S_i}^2 M_P^{2k_i-6}]^{\frac{1}{2k_i-4}}$, which is above GUT scale for larger values of the integer k_i (which in turn is determined by the Z_n symmetry charge of s_i). One could also have large vevs for s_1, s_2 by using anomalous $U(1)$ charges for them using D-terms to break the $U(1)$ symmetry.

The effective theory below the scales $M_{1,2}$ and $\langle s_i \rangle$ of the vector-like pair masses and the s_i-vevs respectively is given by:

$$W = (\phi_1\psi)(\phi_1\psi)H + (\phi_2\psi)(\phi_2\psi)\bar{\Delta} + (\phi_3\psi\psi)\bar{\Delta} + (\phi_2\psi\psi)H', \tag{11}$$

where we have omitted the dimensional coupling constants to make it simple for the purpose of writing. The discrete symmetries prevent ϕ^2/M^2 corrections to these terms. So our predictions based on this effective superpotential do not receive large corrections. We note that the non-renormalizable terms in Eq.(9) can also be obtained from renormalizable couplings if we introduce further S_4-triplet vectorlike fields. Here, however we use only S_4-singlet vectorlike fields to get rank 1 contribution to h and f Yukawa couplings and that is why we need the non-renormalizable terms to be present in Eq.(9).

In order to get fermion masses, we have to find the alignment [12] of the vevs of the flavon fields $\phi_{1,2,3}$. We show below that the following choice of vevs are among the minima of the flavon superpotential provided the couplings of mixed terms between different ϕ_i's are small compared to other couplings:

$$\phi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{12}$$

Clearly, there are other vacua for the flavon model that we do not choose. What is however nontrivial is that the alignments are along quantized directions. This is a consequence of supersymmetry combined with discrete symmetries in the theory. Given these vev, we find from Eq. (11) that the Yukawa coupling matrices h, f, h' have the form:

$$h \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{13}$$

$$f \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \tag{14}$$

$$h' \propto \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{15}$$

and the charged fermion mass matrices can then be inferred. The neutrino mass matrix in this basis has the form:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & c & c \\ c & a & c-a \\ c & c-a & a \end{pmatrix}, \tag{16}$$

where $c/a = \lambda \ll 1$. It is diagonalized by the tri-bi-maximal matrix

$$U_{\mathrm{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \tag{17}$$

This is however not the full PMNS matrix which will receive small corrections from diagonalization of the charged lepton matrix, which not only make small contributions to the θ_{atm} and $\theta_\odot$ but also generate a small θ_{13}.

The neutrino masses are given by $m_{\nu3} = 2a - c$; $m_{\nu2} = 2c$ and $m_{\nu1} = -c$. To fit observations, we require $\lambda = c/a \simeq \sqrt{\Delta m_\odot^2/\Delta m_{\mathrm{atm}}^2} \sim 0.2$, which fixes the neutrino masses $m_{\nu3} \simeq 0.05$ eV, $m_{\nu2} \simeq 0.01$ eV, and $m_{\nu1} \simeq 0.005$ eV. We will see below that λ is also the Cabibbo angle substantiating our claim that neutrino mass ratio and Cabibbo angle are related.

For the charged lepton, up and down quark mass matrices, we have:

$$M_\ell = \frac{r_1}{\tan\beta} \begin{pmatrix} 0 & -3m_1 + \delta & -3m_1 - \delta \\ -3m_1 + \delta & -3m_0 & 3m_0 - 3m_1 \\ -3m_1 - \delta & 3m_0 - 3m_1 & -3m_0 + M \end{pmatrix}, \tag{18}$$

$$M_d = \frac{r_1}{\tan\beta} \begin{pmatrix} 0 & m_1 + \delta & m_1 - \delta \\ m_1 + \delta & m_0 & -m_0 + m_1 \\ m_1 - \delta & -m_0 + m_1 & m_0 + M \end{pmatrix},$$

$$
M_u = \begin{pmatrix} 0 & r_2 m_1 + r_3 \delta & r_2 m_1 - r_3 \delta \\ r_2 m_1 + r_3 \delta & r_2 m_0 & -r_2 m_0 + r_3 m_1 \\ r_2 m_1 + r_3 \delta & -r_2 m_0 + r_3 m_1 & r_2 m_0 + M \end{pmatrix},
$$

where $\tan \beta$ is a ratio of $H_{u,d}$ vevs. Note that $m_1/m_0 = \lambda \sim 0.2$ and of course $m_0 \ll M$. A quick examination of these mass matrices leads to several immediate conclusions:

1. The model predicts that at GUT scale $m_b \simeq m_\tau$.

2. Since $(M_d)_{11} \to 0$, we get $V_{us} \simeq \sqrt{m_d/m_s}$.

3. The empirically satisfied relation $m_\mu m_e \simeq m_s m_d$ can be obtained by the choice of parameters $-3m_1 + \delta = (m_1 + \delta)e^{i\sigma}$, where σ is a phase. Solving this equation, we find that $\delta = m_1(1 + i \cot \sigma/2)$. We obtain $V_{us} \simeq (1 - r_3/r_2)\delta/m_0$, thereby relating Cabibbo angle to the neutrino mass ratio $m_\odot/m_{\text{atm}} \simeq \lambda$.

4. $m_\mu \sim -3m_s$.

5. The leptonic mixing angle to diagonalize M_ℓ is related to quark mixing $\theta_{12}^l \sim \frac{1}{3}V_{us}$, which leads to a prediction for $\sin \theta_{13} \equiv U_{e3} \sim \frac{V_{us}}{3\sqrt{2}} \simeq 0.05$ [11].

6. $V_{cb} \sim \dfrac{m_s}{m_b} \cot \theta_{\text{atm}}$.

7. The masses of up and charm quarks are given by the parameters $r_{2,3}$ and are therefore not predictions of the model.

8. CP violation in quark sector can put in by making the parameters h' complex.

9. The model predicts a small amplitude for neutrino-less double beta decay from light neutrino mass: $m_{\nu_{ee}} \sim c \sin \theta_{12}^l \simeq 0.3$ meV.

The first four relations are fairly well satisfied by observations; the fifth prediction (i.e. that for U_{e3}) can be tested in upcoming reactor and long baseline experiments. Note that the deviation from tri-bi-maximal mixing pattern coming from the charged lepton mass diagonalization could be thought of as a small perturbation of the neutrino mass matrix except that we predict the form of the perturbation from symmetry considerations. The sixth prediction gives a smaller value for V_{cb} (0.02 as against observed GUT scale value of 0.03) if one uses GUT scale extrapolated value of the known b mass. However, in the MSSM there are threshold corrections to the $b-s$ quark mass mixing from gluino and wino exchange one-loop diagrams; by choosing this contribution, one could obtain the desired V_{cb}.

Note that in this model, the top quark Yukawa coupling at GUT scale arises from an effective higher dimensional operator. We have showed the effective operator in Eq.(11) by expanding ϕ/M. The more precise form for the top Yukawa coupling is $\phi^2/(M_1^2+\phi^2)h_{\psi_V\psi_V H}$, where $h_{\psi_V\psi_V H}$ is a coupling of $\psi_V\psi_V H$ term, and ϕ is the vev of ϕ_1 multiplied by $\phi_1\psi\bar{\psi}_V$ coupling. This is simply because the low energy third generation field is a linear combination of the form $\cos\alpha\,\psi_3 - \sin\alpha\,\psi_V$ with the mixing angle $\sin\alpha \simeq \phi/\sqrt{M_1^2+\phi^2}$.

Therefore, in general, there is no gross contradiction to the fact that the top Yukawa coupling is order 1. However, in our case, if ϕ/M_1 becomes close to 1, the atmospheric mixing shifts from the maximal angle. Given the error in the determination of the atmospheric mixing angle, this is consistent with data and as this measurement sharpens, this is going to provide a test of this particular model. The desired smallness of the effective f and h' couplings however are more naturally obtained due to the presence of the Planck mass in the denominator. In order to make the f-coupling dominate over the h', we have to choose a small coupling for the H' Higgs field in Eq. (4). Similarly the λ term in Eq. (9) is assumed to be small compared to the coefficient of the first matrix.

Thus within these set of assumptions, this model is in good phenomenological agreement with observations. In a more complete theory, these assumptions need to be addressed. We however find it remarkable that despite these shortcomings, the model provides a very useful unification strategy of the diverse quark-lepton mixing patterns.

VII.   VEV ALIGNMENT AS MINIMA OF FLAVON THEORIES

In this section, we give examples of how the minima of flavon theories can determine the Yukawa couplings of the fermions and lead to predictive flavor models. We discuss the specific case of the S_4 model at hand. This mechanism is of course applicable to any general group.

We start our discussion by giving some simple examples and discussing the flavon alignment as a prelude to the more realistic example. First thing to note is that $\mathbf{3_1}^3$ is invariant under S_4, but $\mathbf{3_2}^3$ is not. Denoting $\phi = (x,y,z)$, we see that in the first case, the singlet of $\phi^3 = xyz$. The superpotential for a $\mathbf{3_1}$ flavon field ϕ can therefore be written as

$$W = \frac{1}{2}m\phi^2 - \lambda\phi^3 = \frac{1}{2}m(x^2 + y^2 + z^2) - \lambda xyz. \tag{19}$$

The solution of F-flat vacua ($\phi \neq 0$) are

$$\phi = \frac{m}{\lambda}\{(1,1,1) \text{ or } (1,-1,-1) \text{ or } (-1,1,-1) \text{ or } (-1,-1,1)\}. \tag{20}$$

These vacua break S_4 down to S_3 and in the process determine the Yukawa couplings.

On the other hand, when $\mathbf{3_2}$ flavon is used (or the cubic term is forbidden by a discrete symmetry), quartic term involving the triplet is crucial for the F-flat vacua. The invariant quartic term ϕ^4 gives two linear combinations of the form $x^4 + y^4 + z^4$ and $x^2 y^2 + y^2 z^2 + z^2 x^2$. This is because they have to be symmetric homogenous terms and invariant under the Klein's group, which is π rotation around the x, y, z axes.

Thus, the superpotential term for $\mathbf{3_2}$ field ϕ is

$$W = \frac{1}{2} m \phi^2 - \frac{\kappa^{(1)}}{M} (\phi^4)_1 - \frac{\kappa^{(2)}}{M} (\phi^4)_2 \tag{21}$$
$$= \frac{1}{2} (x^2 + y^2 + z^2) - \frac{\kappa^{(1)}}{4M} (x^4 + y^4 + z^4) - \frac{\kappa^{(2)}}{2M} (x^2 y^2 + y^2 z^2 + z^2 x^2).$$

The nontrivial F-flat vacua ($\phi \neq 0$) are

$$\phi = \sqrt{\frac{mM}{\kappa^{(1)}}} \, \vec{a}, \quad \sqrt{\frac{mM}{\kappa^{(1)} + 2\kappa^{(2)}}} \, \vec{b}, \quad \sqrt{\frac{mM}{\kappa^{(1)} + \kappa^{(2)}}} \, \vec{c}, \tag{22}$$

where $\vec{a} = (0, 0, \pm 1), (0, \pm 1, 0), (\pm 1, 0, 0), \vec{b} = (\pm 1, \pm 1, \pm 1),$ and $\vec{c} = (0, \pm 1, \pm 1),$ $(\pm 1, \pm 1, 0), (\pm 1, 0, \pm 1).$ We note that these vectors correspond to the axes of the regular hexahedron. The vacua break S_4 down to Z_4, Z_3, and Z_2, respectively. More importantly, the vacuum states in Eq. (12) used in the analysis of fermion masses in the previous section are a subset of the above vacua.

Note that if we add a ϕ^4 term to the superpotential involving the $\mathbf{3_1}$ flavon field, $\vec{a}$ vacuum is possible, in addition to the original $\vec{b}$ vacua. However, $\vec{c}$ vacuum is absent.

VIII. COMMENTS

A complete understanding of flavor is clearly a very ambitious task. Our proposal should be considered as a simple beginning towards a final theory. It should be noted that even though we have considered on SO(10) group, our general unification ansatz (without as much predictivity) in $SU(2)_L \times SU(2)_R \times SU(4)_c$ theories as well and perhaps other groups such as E_6. Similarly, one should explore other flavor models.

A second point of importance is that while we have kept only leading order terms, one should clearly consider higher order corrections to our predictions systematically. In the above mode, we have checked next order corrections and found them to be absent due to the discrete symmetries.

A final source of corrections could come from anomalies in the discrete symmetries, although we expect them to be small[13].

IX. CONCLUSION

In summary, I have discussed a recently proposed ansatz that has the potential to provide a unified description of the diverse quark and lepton flavor. This could provide the first opening into a very difficult problem of particle physics- the problem of flavor. A simple realization of this ansatz is shown to occur within a grand unified SO(10) model with type II seesaw describing the neutrino masses. The successes of that model are that it seems to provide an understanding of several observed quark-lepton mass relations such as bottom-tau mass unification, strange quark-muon mass ratio (1/3) etc. and predicts a value for $\theta_{13} \sim 0.05$ and atmospheric mixing angle different from the maximal value. The model like most grand unified theories of neutrinos predicts a normal hierarchy and observation of inverted hierarchy will therefore rule out this model (as well as most grand unified theories). Under certain reasonable approximations, this also seems to explain why $m_{solar}/m_{atm} \sim \theta_C$. Both the predictions given above (θ_{13} and θ_{atm}) could be used to test the model in the upcoming long baseline neutrino experiments. It also predicts a value of 0.3 meV for the neutrinoless double beta decay experiments.

---

[1] S. Weinberg, Trans. New York Acad. Sci. **38**, 185 (1977); F. Wilczek and A. Zee, Phys. Lett. B **70**, 418 (1977) [Erratum-ibid. **72B**, 504 (1978)]; H. Fritzsch, Phys. Lett. B **73**, 317 (1978).

[2] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002) [hep-ph/0202074]; X. G. He and A. Zee, Mod. Phys. Lett. A **22**, 2107 (2007) [arXiv:hep-ph/0702133]; Z. z. Xing, Phys. Lett. B **533**, 85 (2002) [hep-ph/0204049].

[3] P. Minkowski, Phys. Lett. B **67** (1977) 421; T. Yanagida in *Workshop on Unified Theories, KEK Report 79-18*, p. 95, 1979; M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity*, p. 315. Amsterdam: North Holland, 1979; S. L. Glashow, *1979 Cargese Summer Institute on*

Quarks and Leptons, p. 687. New York: Plenum, 1980; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett, **44** (1980) 912.

[4] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. D **80**, 095021 (2009); JHEP **1005**, 034 (2010).

[5] B. S. Balakrishna, A. L. Kagan and R. N. Mohapatra, Phys. Lett. B **205**, 345 (1988); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **64**, 2747 (1990); B. A. Dobrescu and P. J. Fox, JHEP **0808**, 100 (2008); L. Ferretti, S. F. King and A. Romanino, JHEP **0611**, 078 (2006).

[6] K. S. Babu, B. Dutta and R. N. Mohapatra, Phys. Rev. D **60**, 095004 (1999).

[7] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).

[8] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

[9] B. Bajc, G. Senjanovic and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003).

[10] G. Altarelli and F. Feruglio, Nucl. Phys. B **741**, 215 (2006) [hep-ph/0512103]; G. Altarelli, F. Feruglio and C. Hagedorn, JHEP **0803**, 052 (2008) [arXiv:0802.0090 [hep-ph]]; F. Feruglio, C. Hagedorn and L. Merlo, JHEP **1003**, 084 (2010) [arXiv:0910.4058 [hep-ph]]; S. Morisi and E. Peinado, Phys. Rev. D **80**, 113011 (2009) [arXiv:0910.4389 [hep-ph]]; M. C. Chen and S. F. King, JHEP **0906**, 072 (2009) [arXiv:0903.0125 [hep-ph]]; Y. Cai and H. B. Yu, Phys. Rev. D **74**, 115005 (2006) [hep-ph/0608022]; F. Bazzocchi and S. Morisi, Phys. Rev. D **80**, 096005 (2009) [arXiv:0811.0345 [hep-ph]]; H. Ishimori, Y. Shimizu and M. Tanimoto, Prog. Theor. Phys. **121**, 769 (2009) [arXiv:0812.5031 [hep-ph]]; F. Bazzocchi, L. Merlo and S. Morisi, Phys. Rev. D **80**, 053003 (2009) [arXiv:0902.2849 [hep-ph]]; G. Altarelli, F. Feruglio and L. Merlo, JHEP **0905**, 020 (2009) [arXiv:0903.1940 [hep-ph]]; W. Grimus, L. Lavoura and P. O. Ludl, J. Phys. G **36**, 115007 (2009) [arXiv:0906.2689 [hep-ph]]; G. J. Ding, Nucl. Phys. B **827**, 82 (2010) [arXiv:0909.2210 [hep-ph]]; Y. Daikoku and H. Okada, arXiv:0910.3370 [hep-ph]; M. K. Parida, Phys. Rev. D **78**, 053004 (2008) [arXiv:0804.4571 [hep-ph]]; C. Luhn, S. Nasri and P. Ramond, Phys. Lett. B **652**, 27 (2007) [arXiv:0706.2341 [hep-ph]]; C. Hagedorn, M. A. Schmidt and A. Y. Smirnov, Phys. Rev. D **79**, 036002 (2009) [arXiv:0811.2955 [hep-ph]]; F. Bazzocchi and I. de Medeiros Varzielas, Phys. Rev. D **79**, 093001 (2009) [arXiv:0902.3250 [hep-ph]]; A. Adulpravitchai, A. Blum and C. Hagedorn, JHEP **0903**, 046

(2009) [arXiv:0812.3799 [hep-ph]]; S. F. King and C. Luhn, Nucl. Phys. B **820**, 269 (2009) [arXiv:0905.1686 [hep-ph]]; M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **681**, 444 (2009) [arXiv:0904.1721 [hep-ph]]; arXiv:0910.5467 [hep-ph]; For some earlier models, see K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003) [hep-ph/0206292]; K. S. Babu and X. G. He, hep-ph/0507217; R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B **639**, 318 (2006) [hep-ph/0605020].

For recent reviews, see G. Altarelli and F. Feruglio, arXiv:1002.0211 [hep-ph]; H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, arXiv:1003.3552 [hep-th].

[11] J. Ferrandis and S. Pakvasa, Phys. Lett. B **603**, 184 (2004) [hep-ph/0409204]; S. F. King, JHEP **0508**, 105 (2005) [hep-ph/0506297]; B. Dutta and Y. Mimura, Phys. Lett. B **633**, 761 (2006) [hep-ph/0512171]; M. C. Chen and K. T. Mahanthappa, Phys. Lett. B **652**, 34 (2007).

[12] S. F. King and G. G. Ross, Phys. Lett. B **520**, 243 (2001) [hep-ph/0108112]; S. F. King, JHEP **0508**, 105 (2005) [hep-ph/0506297]; I. de Medeiros Varzielas and G. G. Ross, Nucl. Phys. B **733**, 31 (2006) [hep-ph/0507176]; I. de Medeiros Varzielas, S. F. King and G. G. Ross, Phys. Lett. B **644**, 153 (2007) [hep-ph/0512313]; Phys. Lett. B **648**, 201 (2007) [hep-ph/0607045]. S. F. King and C. Luhn, Nucl. Phys. B **832**, 414 (2010); C. Hagedorn, S. F. King and C. Luhn, JHEP **1006**, 048 (2010).

[13] M. Ratz, private communications. R. Kappl, H. P. Nilles, S. Ramos-Sanchez, M. Ratz, K. Schmidt-Hoberg and P. K. S. Vaudrevange, Phys. Rev. Lett. **102**, 121602 (2009)